was used to detect the noise. Between pulses there was no discernible noise, as expected. Initially, only a very sharp spike of -70 dbm amplitude and a duration of a fraction of a tenth of a microsecond was detected approximately 0.05 μsec after the switch tube was triggered. The sharp spike appeared to be generated by the ionization process of the discharge, and may be due to traveling space potential as described by Westberg.[10]

At first no other noise could be detected, indicating that the switch tube noise was at least 10 db below the noise tube. As the tube aged, noise became discernible in other parts of the pulse and increased monotonically until the cathode ceased functioning. The increased noise generation appears to be due to deposition of cathode material around the tube and on the grid. Secondary electron emission from the grid and other parts of the tube can take place by positive ion bombardment. It may be possible that thermionic emission from the grid may occur. These and the unstable cathode emission probably contributed to the noise output of the tube.

[10] R. G. Westberg, "Nature and role of ionizing potential space waves in glow to arc transitions," *Phys. Rev.*, vol. 114, pp. 1–17; April, 1959.

Conclusion

Low-pressure, hot cathode arc discharges offer considerable promise for the development of rapid, broadband microwave switches and control devices. Plasma densities of the order of 10^{13} per cubic centimeter can be achieved in fractions of a microsecond and dissipated in a few microseconds or less. The power level at which microwave capture of the switch occurs may be adjusted by controlling gas pressure and geometry. Switching action for switching a pulsed source is provided by a low level trigger (10–100 volts) which discharges a storage condenser through the switch tube.

Characteristic Impedances of Broadside-Coupled Strip Transmission Lines*

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Summary—**Formulas are given for the even- and odd-mode characteristic impedances of shielded coupled strip-transmission-line configurations that are especially useful when close coupling is desired. Applications may be made to wideband coupled-strip-line filters, 3-db directional couplers, and many other components. The cross sections considered are thin broadside-coupled strips either parallel or perpendicular to the ground planes. Modification of the formulas for thick strips is discussed. The derivations are outlined, with particular attention given to the underlying assumption that restricts the use of the formulas to cases of close coupling.**

I. Introduction

Coupling effects between parallel transmission lines have applications in the design of many components, such as filters,[1,2] directional couplers,[1,3,4] baluns,[5] and differential-phase-shift networks. Three useful coupled-strip-line configurations are shown in Fig. 1. The coplanar-strip cross section of Fig. 1(a) was analyzed previously and design data are conveniently available,[7] while the broadside-coupled strip-line cross sections of Fig. 1(b) and 1(c) are treated in this paper. In all three cases, the theory applies to strips of zero thickness, which may be approximated by metal-foil conductors sandwiched between dielectric plates filling the cross section. If air dielectric is desired, the strips must be given a moderate thickness to provide

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[1] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 75–81; April, 1956.

[2] S. B. Cohn, "Parallel-coupled transmission-line-resonator filters," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 224–231; April, 1958.

[3] J. K. Shimizu, "Strip-line 3-db directional couplers," 1957 IRE WESCON Convention Record, pt. 1, pp. 4–15.

[4] J. K. Shimizu and E. M. T. Jones, "Coupled-transmission-line directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 403–410; October, 1958.

[5] E. M. T. Jones and J. K. Shimizu, "A wide-band strip-line balun," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 128–134; January, 1959.

[6] B. M. Schiffman, "A new class of broad-band microwave 90 degree phase shifters," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 232–237; April, 1958.

[7] S. B. Cohn, "Shielded coupled-strip transmission lines," IRE Trans. on Microwave Theory and Techniques, vol. MTT-3, pp. 29–38; October, 1955.

mechanical strength. In this case, adequate computational accuracy may be obtained by suitably modifying the zero-thickness formulas.

The coplanar configuration of Fig. 1(a) is particularly well suited to photo-etched strip-line circuits, but cannot be used where very close coupling is required. For example, in directional couplers having couplings greater than about -8 db, the spacing between the strips becomes prohibitively small. However, closer coupling may be obtained with a reasonable spacing if thick strips are used in the arrangement of Fig. 1(a). This is clear if one considers that a sizable parallel-plate capacitance is added to the fringing capacitance between the two strips. To achieve close coupling with thin strips, a broadside coupling arrangement as in Fig. 1(b) and 1(c) is necessary. Both of these configurations have been used successfully in directional couplers having midband couplings as great as -2.7 db.[3]

The details of the analysis of the two broadside-coupled strip configurations are not given here, since they involve a fairly routine application of the well-known Schwartz-Christoffel transformation method. However, the specific transformation geometries are described in Section IV, where they reveal a simplifying approximation that was made in each derivation. It is believed that the error in computed values of characteristic impedance due to these approximations will not be greater than about 0.1 per cent in close-coupling applications. The error increases as the coupling is decreased, but is not likely to exceed 1 per cent in any practical application for which a broadside-coupled strip arrangement would be preferred over a coplanar arrangement. The advantage gained by these approximations is that the elliptic functions necessary in exact solutions are avoided, thus simplifying the use of the formulas, as well as their derivation. However, the reader will be interested to know that rigorous solutions valid for all values of the parameters have recently been carried out by Hachemeister.[8]

II. Formulas for Broadside-Coupled Strips Parallel to Ground Planes

Two orthogonal TEM modes can propagate on a pair of parallel-coupled transmission lines. These are the *even* mode, for which the respective voltages and currents on the two conductors are equal and of the same sign, and the *odd* mode, for which the respective voltages and currents are equal but of opposite sign. All other voltage, current, and field conditions on the conductors can be expressed as a linear combination of these two modes and, therefore, the complete performance of the coupled conductors in a circuit may be computed in terms of the characteristic impedances and propagation constants of these modes. It will be assumed that losses are small and that the cross sections are uniformly filled with a medium of relative dielectric constant ϵ_r, so that the characteristic impedances are essentially real, the attenuation is small, and the phase velocities are equal to $c/\sqrt{\epsilon_r}$, where c is the velocity of light in free space.

The following formulas give the even-mode characteristic impedance, Z_{oe}, and the odd-mode characteristic impedance, Z_{oo}, of each conductor with respect to ground for the cross section of Fig. 2. These formulas hold for any ratio of w/b and s/b, as long as w/s is greater than about 0.35:

$$Z_{oe} = \frac{188.3}{\sqrt{\epsilon_r}} \frac{K(k')}{K(k)} \tag{1}$$

$$Z_{oo} = \frac{296.1}{\sqrt{\epsilon_r}\, \frac{b}{s} \tanh^{-1} k} \tag{2}$$

where

$k =$ a parameter,
$k' = \sqrt{1-k^2}$,
$K(k)$ and $K(k')$ = complete elliptic integrals of the first kind.

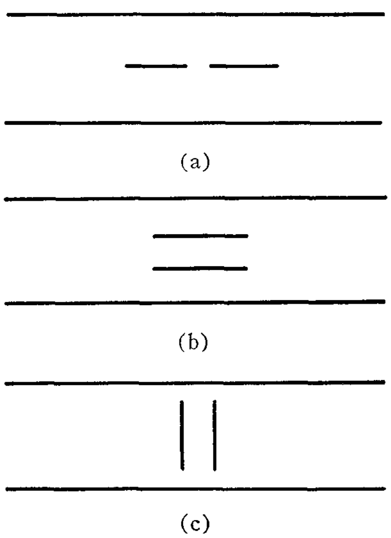

Fig. 1—Three useful configurations of very thin coupled strips between parallel ground planes.

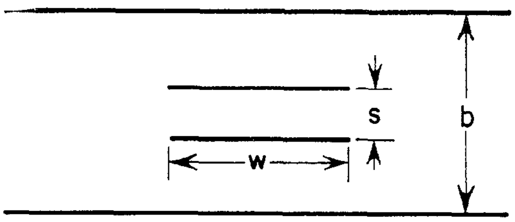

Fig. 2—Cross-section dimensions of broadside-coupled strips parallel to the ground planes.

[8] C. A. Hachemeister, "The Impedances and Fields of Some TEM-Mode Transmission Lines," Microwave Res. Inst., Polytechnic Inst. of Brooklyn, N. Y., Res. Rept. No. R-623-57, PIB-551, Contract No. AF 19(604)-2031, ASTIA Doc. No. AD-160802; April 16, 1958.

The ratio w/b is given by

$$\frac{w}{b} = \frac{2}{\pi}\left\{\tanh^{-1}\sqrt{\frac{k\dfrac{b}{s}-1}{\dfrac{1}{k}\dfrac{b}{s}-1}}\right.$$

$$\left. - \frac{s}{b}\tanh^{-1}\left[\frac{1}{k}\sqrt{\frac{k\dfrac{b}{s}-1}{\dfrac{1}{k}\dfrac{b}{s}-1}}\right]\right\}. \qquad (3)$$

The quantity $K(k')/K(k)$ has been tabulated vs k by Oberhettinger and Magnus.[9] With the aid of their table, a cross section may be designed in a straightforward process to have the desired characteristic impedances Z_{oe} and Z_{oo}, as follows. First, from (1), solve for $K(k')/K(k)$. Next determine k from the abovementioned table.[10] Then, obtain b/s from (2). Finally, calculate w/b from (3).

A. Simplification for $(w/b)/(1-s/b) \geq 0.35$

If, in addition to the condition $w/s \geq 0.35$, one imposes the condition

$$\frac{w/b}{1 - s/b} \geq 0.35,$$

the fringing fields at opposite edges of the strips will be sufficiently isolated that fringing capacitances can be specified that are essentially independent of the strip width. Then, the characteristic impedances are given by

$$Z_{oe} = \frac{188.3/\sqrt{\epsilon_r}}{\dfrac{w/b}{1-s/b} + \dfrac{C_{fe}'}{\epsilon}} \qquad (4)$$

and

$$Z_{oo} = \frac{188.3/\sqrt{\epsilon_r}}{\dfrac{w/b}{1-s/b} + \dfrac{w}{s} + \dfrac{C_{fo}'}{\epsilon}}, \qquad (5)$$

where the fringing capacitance, C_{fe}', is the capacitance per unit length that must be added at each edge of each strip to the parallel-plate capacitance so that the total capacitance to ground for the even-mode field distribution will be correct. C_{fo}' is the corresponding quantity for the odd mode, and ϵ is the absolute dielectric constant, equal to 8.85 ϵ_r mmf per meter. The fringing capacitances are functions only of s/b, and are given by

$$\frac{C_{fe}'}{\epsilon} = 0.4413 + \frac{1}{\pi}\left[\log_e\left(\frac{1}{1-s/b}\right) + \frac{s/b}{1-s/b}\log_e\frac{b}{s}\right] \qquad (6)$$

$$\frac{C_{fo}'}{\epsilon} = \frac{b/s}{\pi}\left[\log_e\left(\frac{1}{1-s/b}\right) + \frac{s/b}{1-s/b}\log_e\frac{b}{s}\right]. \qquad (7)$$

Eqs. (6) and (7) are plotted vs s/b in Fig. 3.

B. Effect of a Small Thickness of the Strips

When foil strips are used in the cross section of Fig. 2, the strip thickness of about 0.0015 inch is usually not great enough to affect seriously the zero-thickness values of C_{fe}'/ϵ and C_{fo}'/ϵ, but it can have an appreciable effect on the parallel-plate capacitance between the strips in the odd-mode case, depending on how the dimension s is defined. In the usual case of $s/b<0.5$, s should be taken to be the spacing between the strips, as in Fig. 4; then (1)–(3) or (4)–(7) may be used with good accuracy. For greater strip thickness, the fringing capacitances will be larger than the values given by (6) and (7) and Fig. 3. The parallel-plate capacitance between each strip and its adjacent ground plane will also be increased. A paper on thickness corrections that

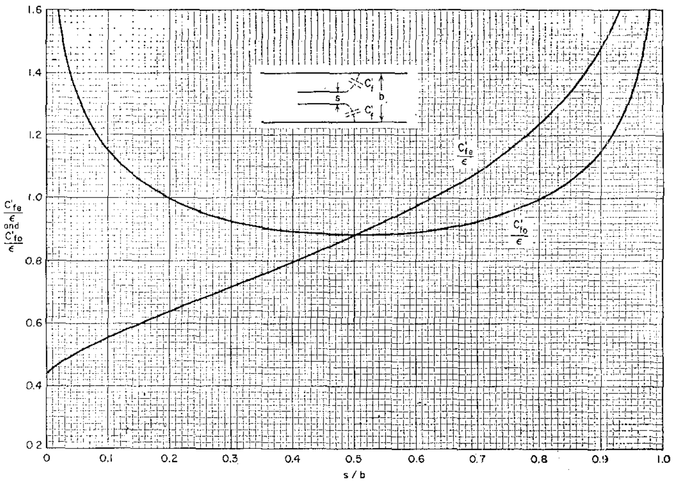

Fig. 3—Even- and odd-mode fringing capacitances for broadside-coupled very thin strips parallel to the ground planes.

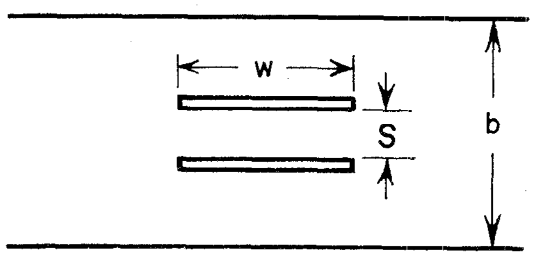

Fig. 4—Thick strips in the broadside-coupled parallel arrangement.

[9] F. Oberhettinger and W. Magnus, "Anwendung der Elliptischen Functionen in Physik und Technik," Springer-Verlag, Berlin, Germany; 1949.

[10] For k small, one may use

$$\frac{K(k')}{K(k)} = \frac{2}{\pi}\log_e\frac{4}{k}.$$

The error in this formula increases with k, being about $\frac{1}{4}$ per cent for $k=0.2$, and 1 per cent for $k=0.3$.

applies to this and numerous other practical configurations is published in this issue.[11]

III. Formulas for Broadside-Coupled Strips Perpendicular to Ground Planes

The even- and odd-mode characteristic impedances of the cross section of Fig. 5 are given by these formulas:

$$Z_{oe} = \frac{188.3}{\sqrt{\epsilon_r}} \frac{K(k)}{K(k')} \tag{8}$$

$$Z_{oo} = \frac{296.1/\sqrt{\epsilon_r}}{\frac{b}{s}\cos^{-1} k + \log_e \frac{1}{k}} \tag{9}$$

where k is a parameter and k', $K(k)$, and $K(k')$ are as defined in Section II. The ratio w/b is given by

$$\frac{w}{b} = \frac{2}{\pi}\left\{\tan^{-1}\left[\frac{k'}{k}\sqrt{\frac{1-\frac{k}{k'}\frac{s}{b}}{1+\frac{k'}{k}\frac{s}{b}}}\right] - \frac{s}{b}\tanh^{-1}\left[\sqrt{\frac{1-\frac{k}{k'}\frac{s}{b}}{1+\frac{k'}{k}\frac{s}{b}}}\right]\right\}. \tag{10}$$

The inverse cosine and tangent functions in (9) and (10) are evaluated in radians between 0 and $\pi/2$. As in the case of broadside strips parallel to the ground planes, the dimension ratios may be determined in a straightforward manner for given values of Z_{oe} and Z_{oo}. With the aid of the table of $K(k)/K(k')$ vs k in Oberhettinger and Magnus,[9] the parameter k is determined from (8). Then b/s is obtained from (9), and finally w/b from (10).

Eqs. (8)–(10) are accurate for all values of w/b and s/b, as long as w/s is greater than about 1.0. As s/b is made small compared to 1.0, the formulas remain accurate for smaller values of w/s. This range of validity covers all practical cases except that of very loose coupling.

A. Effect of a Small Thickness of the Strips

In the case of foil strips, the effect of strip thickness can be neglected if s is defined to be the spacing between the strips, as shown in Fig. 6. For thicker strips the formulas should be used with discretion. A companion paper on thickness corrections, referred to in Section II, will enable an improvement in accuracy in the case of thick strips.[11]

[11] S. B. Cohn, "Thickness corrections for capacitive obstacles and strip conductors," this issue, p. 638.

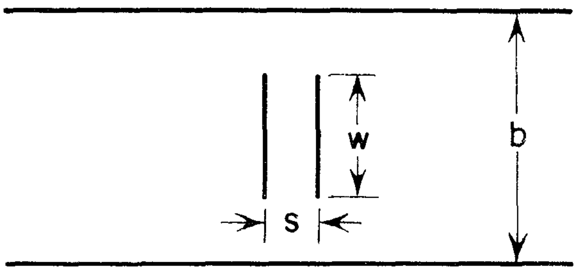

Fig. 5—Cross-section dimensions of broadside-coupled strips perpendicular to the ground planes.

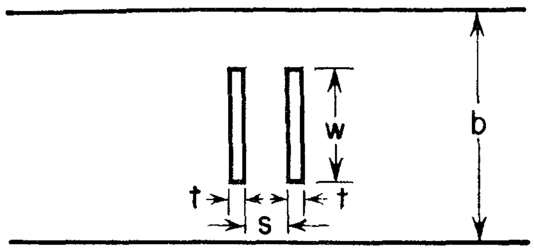

Fig. 6—Thick strips in the broadside-coupled, perpendicular arrangement.

IV. Comments on the Derivations

The formulas in this paper were derived by the Schwartz-Christoffel conformal-transformation method.[12,13] This method enables one to evaluate the capacitance and characteristic impedance between straight-sided conductors when the problem can be reduced to two dimensions, as in the cross-section plane of a transmission line. By means of one or more transformations in the complex plane, the boundary of the cross section is transformed into a simpler boundary for which the solution is known. Then because of the special properties of the conformal transformation, the capacitance and characteristic impedance of the original boundary are equal to the respective quantities of the transformed boundary.

Fig. 7(a) shows the cross section of broadside-coupled strips parallel to the ground planes. Vertical and horizontal planes of symmetry are indicated, and because of this symmetry the solution can be obtained from the geometry of Fig. 7(b), which is the upper right-hand quarter of the complete cross section. The boundary in Fig. 7(b) consists of half of the upper ground plane, the two sides of half of the upper strip, two vertical magnetic-wall segments, and a horizontal plane which is a magnetic wall for the even-mode case and an electric wall for the odd-mode case. In addition, the open end of the boundary is assumed to be closed at infinity. An exact solution of Fig. 7(b) requires elliptic functions. In order to avoid these functions, it is necessary to reduce the number of right-angle corners in the boundary. One way of doing this is shown in Fig. 7(c), where the lower

[12] W. R. Smythe, "Static and Dynamic Electricity," McGraw-Hill Book Co., Inc., New York, N. Y., 1st ed., p. 80 ff.; 1939.
[13] E. Weber, "Electromagnetic Fields," vol. 1, John Wiley and Sons, Inc., New York, N. Y., p. 325 ff.; 1950.

vertical segment of magnetic wall is shifted to minus infinity. The new boundary is not exactly equivalent to Fig. 7(a) but the approximation is excellent in the usual case of $w > s$. In that case it is clear that for the even mode very little electric-field energy penetrates into the region on the left of the $x=0$ plane in the lower left-hand part of Fig. 7(c), while for the odd mode, the electric field is virtually uniform in that region. Thus very good accuracy is obtained in the even-mode case by evaluating the total capacitance between the conductors of Fig. 7(c) and assuming it to be equal to the capacitance of Fig. 7(b), and in the odd-mode case by calculating the total capacitance minus the parallel-plate capacitance between $x=0$ and $x=-\infty$. This procedure has been followed in the derivation of (1)–(3), and no further approximations were made. Experience with other geometries of strip conductors indicates that an accuracy of the order of 1 per cent should ensue for w/s as low as 0.35, and of the order of 0.1 per cent for w/s greater than 1.0.

A similar approximation was made in the analysis of broadside-coupled strips perpendicular to the ground planes. The cross section is shown with its symmetry planes in Fig. 8(a), and the exact quarter model is shown in Fig. 8(b). As in the previous case, elliptic functions are needed in the solution of Fig. 8(b), but are avoided in the solution of Fig. 8(c), which is the geometry that was used in the derivation of (8)–(10). The total even-mode capacitance in Fig. 8(c) is assumed equal to the even-mode capacitance in Fig. 8(b), while the total odd-mode capacitance minus the parallel plate capacitance from $y=0$ to $y=-\infty$ is assumed equal to the odd-mode capacitance in Fig. 8(b). The considerations about accuracy are the same as in the previous case, but it is believed that w/s should not be made smaller than 1.0 unless further evidence justifies a lower limit.

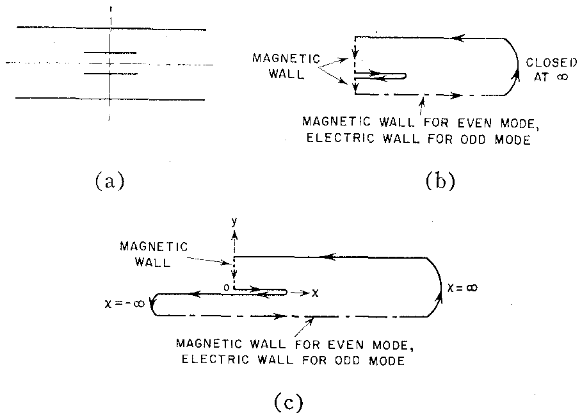

Fig. 7—Boundaries considered for the broadside-coupled parallel case.

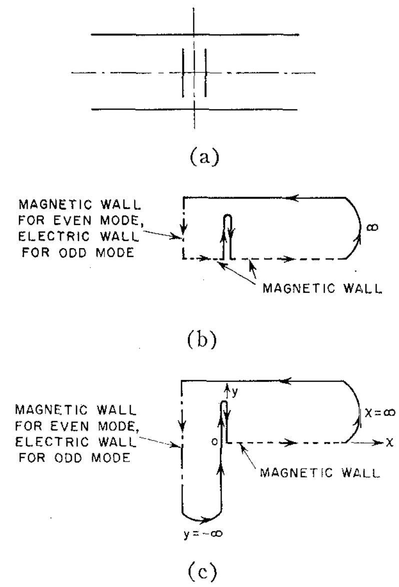

Fig. 8—Boundaries considered for the broadside-coupled perpendicular case.